Slip Effects on the Magnetohydrodynamic Peristaltic Flow of a Maxwell Fluid

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The influence of slip on the magnetohydrodynamic (MHD) peristaltic flow in a planar channel with compliant walls is examined. An incompressible Maxwell fluid saturates the porous medium. An established solution is valid for small wave number. The mathematical expression of the stream function is presented. Several interesting flow parameters are sketched and examined.

Key words: Maxwell Fluid; Compliant Walls; Slip Effects.

1. Introduction

The peristaltic flows in tubes/channels are of paramount importance in physiological systems. These include urine transport from kidney to bladder, chyme motion in the gastrointestinal tract, blood circulation in small blood vessels, and many others. Besides these such flows occur in blood pumps in heart lung machine, in sanitary fluid transport, and transport of corrosive fluids. The pioneering works on this topic have been initiated by Latham [1] and Jaffrin and Shapiro [2]. The peristaltic transport of a Maxwell fluid in the presence of a Hall current and in a porous medium has been examined by Hayat et al. [3]. In another investigation, Wang et al. [4] analyzed the slip effects on the peristaltic flow of a third-grade fluid in circular cylinder. The heat transfer effects on the peristaltic flow of viscous fluid in a channel have been studied by Radhakrishnamacharya and Srinivasulu [5]. The effect of wall properties on peristaltic transport of a viscous fluid have been seen by Abd Elnaby and Haroun [6]. The slip effects on the peristaltic flow of a third grade fluid in an asymmetric channel are discussed by Hayat et al. [7]. The heat transfer on the MHD peristaltic flow filling a porous space is studied by Kothandapani and Srinivas [8]. The compliant wall effects on the peristaltic flow of a Johnson-Segalman fluid in a channel are examined by Hayat et al. [9]. Hayat et al. [10] also performed a study for peristaltic flow of a Maxwell fluid in an asymmetric channel by considering the small wave number. Hayat and Hina [11] discussed the influence heat and mass transfer on the MHD peristaltic flow of a Maxwell fluid in a porous medium for small wave number. Ellahi [12] examined the effects of slip on the non-Newtonian flow in a channel. Slip effects on the nonlinear flows of a third-grade fluid have been discussed by Ellahi et al. [13]. In another paper, Ellahi et al. [14], studied the flow of an Oldroyd 8-constant fluid subject to slip boundary conditions.

Existing literature shows that the slip effects on MHD peristaltic transport of a non-Newtonian fluid in a porous space is not discussed so far. Hence, the main theme of the present attempt is to discuss the slip effects on the MHD peristaltic flow of a Maxwell fluid in a porous space. The slip condition in terms of shear stress has been accounted. The mathematical formulation of the problem is based upon the modified Darcy law. The series solution is derived for small wave number and the salient features of the embedded parameters are pointed out.

2. Formulation of the Problem

We discuss an incompressible Maxwell fluid of density ρ and kinematic viscosity ν in a planar channel of width $2d_1$. An incompressible and electrically conducting fluid is considered. A uniform applied magnetic field $\mathbf{B}_0$ acts in the y-direction (the x- and y-axes are chosen along and perpendicular to the channel walls, respectively). The flow is caused by imposed sinusoidal waves on the compliant walls of the channel.

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The wave shapes are described by the following equation:

$$y = \pm\eta(x,t) = \pm\left[d_1 + a\sin\frac{2\pi}{\lambda}(x-ct)\right], \quad (1)$$

where c is the wave speed, a and λ are the wave amplitude and wavelength, respectively. The basic equations governing the flow are

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \quad (2)$$

$$\rho\left[\frac{\partial}{\partial t} + u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y}\right]u = -\frac{\partial p}{\partial x} + \frac{\partial S_{xx}}{\partial x} + \frac{\partial S_{xy}}{\partial y} - \sigma B_0^2 u + R_x, \quad (3)$$

$$\rho\left[\frac{\partial}{\partial t} + u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y}\right]v = -\frac{\partial p}{\partial y} + \frac{\partial S_{xy}}{\partial x} + \frac{\partial S_{yy}}{\partial y} + R_y, \quad (4)$$

and expressions for the components of extra stress tensor are

$$S_{xx} + \lambda_1\left[\frac{\mathrm{d}S_{xx}}{\mathrm{d}t} - 2\left(S_{xx}\frac{\partial u}{\partial x} + S_{xy}\frac{\partial u}{\partial y}\right)\right] = 2\mu\frac{\partial u}{\partial x}, \quad (5)$$

$$S_{yy} + \lambda_1\left[\frac{\mathrm{d}S_{yy}}{\mathrm{d}t} - 2\left(S_{xy}\frac{\partial v}{\partial x} + S_{yy}\frac{\partial v}{\partial y}\right)\right] = 2\mu\frac{\partial v}{\partial y}, \quad (6)$$

$$S_{xy} + \lambda_1\left[\frac{\mathrm{d}S_{xy}}{\mathrm{d}t} - S_{xx}\frac{\partial v}{\partial x} - S_{yy}\frac{\partial u}{\partial y}\right] = \mu\left[\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right], \quad (7)$$

and the boundary conditions are

$$u \pm \beta S_{xy} = 0 \ \text{ at } \ y = \pm\eta, \quad (8)$$

$$\left[-\tau\frac{\partial^3}{\partial x^3} + m\frac{\partial^3}{\partial x\partial t^2} + d\frac{\partial^2}{\partial t\partial x} + B\frac{\partial^5}{\partial x^5} + H\frac{\partial}{\partial x}\right]\eta = \frac{\partial S_{xx}}{\partial x} + \frac{\partial S_{xy}}{\partial y} - \sigma B_0^2 u + R_x - \rho\frac{\mathrm{d}u}{\mathrm{d}t} \ \text{ at } \ y = \pm\eta. \quad (9)$$

In the above equations u, v denote velocities in x- and y-directions, respectively, p denotes the pressure, σ shows the electrical conductivity of the fluid, β depicts the slip parameter, K indicates the permeability parameter, τ shows the elastic tension in the membrane, m is the mass per unit area, d is the coefficient of viscous damping, B is the flexural rigidity of the plate, H is the spring stiffness, λ_1 is the relaxation time, S_{xx}, S_{xy}, and S_{yy} are the components of the extra stress tensor $\mathbf{S}$, and R_x and R_y are the x- and y-components of the Darcy resistance $\mathbf{R}$.

In a Maxwell fluid the Darcy resistance $\mathbf{R}$ satisfies the following expression [11]:

$$\left(1 + \lambda_1\frac{\mathrm{d}}{\mathrm{d}t}\right)\mathbf{R} = -\frac{\mu}{k}\mathbf{V}. \quad (10)$$

Invoking $\partial/\partial y(5) - \partial/\partial x(6)$ we have

$$\rho\left(1 + \lambda_1\frac{\mathrm{d}}{\mathrm{d}t}\right)\frac{\mathrm{d}}{\mathrm{d}t}\left[\frac{\partial u}{\partial y} - \frac{\partial v}{\partial x}\right] = -\frac{\mu}{k}\left[\frac{\partial u}{\partial y} - \frac{\partial v}{\partial x}\right] + \left(1 + \lambda_1\frac{\mathrm{d}}{\mathrm{d}t}\right) \cdot\left[\frac{\partial}{\partial y}\left\{\frac{\partial S_{xx}}{\partial x} + \frac{\partial S_{xy}}{\partial y} - \sigma B_0^2 u\right\} - \frac{\partial}{\partial x}\left\{\frac{\partial S_{xy}}{\partial x} + \frac{\partial S_{yy}}{\partial y}\right\}\right]. \quad (11)$$

Denoting the stream function $\psi(x,y,t)$ by

$$u = \frac{\partial\psi}{\partial y}, \quad v = -\frac{\partial\psi}{\partial x}$$

and using the following dimensionless variables

$$\psi^* = \frac{\psi}{cd_1}, \quad x^* = \frac{x}{\lambda}, \quad y^* = \frac{y}{d_1}, \quad t^* = \frac{ct}{\lambda},$$
$$\eta^* = \frac{\eta}{d_1}, \quad p^* = \frac{d_1^2 p}{c\lambda\mu}, \quad K = \frac{k}{d^2}, \quad S_{ij}^* = \frac{\mathrm{d}S_{ij}}{c\mu},$$
$$\lambda_1^* = \lambda_1\frac{c}{d_1}, \quad \beta^* = \frac{\mu\beta}{d_1},$$

equations (5) – (9) and (11) reduce to the following expressions:

$$S_{xx} + \lambda_1\left[\delta\left(\frac{\partial}{\partial t} + \psi_y\frac{\partial}{\partial x} - \psi_x\frac{\partial}{\partial y}\right)S_{xx} - 2\left(\delta S_{xx}\psi_{xy} + S_{xy}\psi_{yy}\right)\right] = 2\delta\psi_{xy}, \quad (12)$$

$$S_{yy} + \lambda_1\left[\delta\left(\frac{\partial}{\partial t} + \psi_y\frac{\partial}{\partial x} - \psi_x\frac{\partial}{\partial y}\right)S_{yy} + 2\left(\delta^2 S_{xy}\psi_{xx} + \delta S_{yy}\psi_{xy}\right)\right] = -2\delta\psi_{xy}, \quad (13)$$

$$S_{xy} + \lambda_1\left[\delta\left(\frac{\partial}{\partial t} + \psi_y\frac{\partial}{\partial x} - \psi_x\frac{\partial}{\partial y}\right)S_{xy} + \left(\delta^2 S_{xx}\psi_{xx} - S_{yy}\psi_{yy}\right)\right] = \psi_{yy} - \delta^2\psi_{xx}, \quad (14)$$

$$Re\delta\left[1+\delta\lambda_1\left(\frac{\partial}{\partial t}+\psi_y\frac{\partial}{\partial x}-\psi_x\frac{\partial}{\partial y}\right)\right]\left[\left(\frac{\partial}{\partial t}+\psi_y\frac{\partial}{\partial x}-\psi_x\frac{\partial}{\partial y}\right)\left(\delta^2\frac{\partial^2\psi}{\partial x^2}+\frac{\partial^2\psi}{\partial y^2}\right)\right]=-\frac{1}{K}(\psi_{yy}+\delta^2\psi_{xx})+\left[1+\delta\lambda_1\left(\frac{\partial}{\partial t}+\psi_y\frac{\partial}{\partial x}-\psi_x\frac{\partial}{\partial y}\right)\right]\left[\left(\frac{\partial^2}{\partial y^2}-\delta^2\frac{\partial^2}{\partial x^2}\right)\cdot S_{xy}-M^2\psi_{yy}+\delta\frac{\partial^2}{\partial x\partial y}(S_{xx}-S_{yy})\right], \tag{15}$$

$$\left[1+\delta\lambda_1\left(\frac{\partial}{\partial t}+\psi_y\frac{\partial}{\partial x}-\psi_x\frac{\partial}{\partial y}\right)\right]\left[E_1\frac{\partial^3}{\partial x^3}+E_2\frac{\partial^3}{\partial x\partial t^2}+E_3\frac{\partial^2}{\partial t\partial x}+E_4\frac{\partial^5}{\partial x^5}+E_5\frac{\partial}{\partial x}\right]\eta=\left[1+\delta\lambda_1\left(\frac{\partial}{\partial t}+\psi_y\frac{\partial}{\partial x}-\psi_x\frac{\partial}{\partial y}\right)\right]\left[\delta\frac{\partial S_{xx}}{\partial x}+\frac{\partial S_{xy}}{\partial y}-M^2\psi_y-Re\delta\left(\frac{\partial}{\partial t}+\psi_y\frac{\partial}{\partial x}-\psi_x\frac{\partial}{\partial y}\right)\psi_y\right]-\frac{1}{K}\psi_y \quad \text{at } y=\pm\eta, \tag{16}$$

$$\psi_y\pm\beta S_{xy}=0 \quad \text{at } y=\pm\eta=\pm(1+\varepsilon\sin 2\pi(x-t)). \tag{17}$$

Note that the continuity equation (2) is automatically satisfied, ε $(=a/d_1)$, δ $(=d_1/\lambda)$ are geometric parameters, Re $(=cd_1/\upsilon)$ is the Reynolds number, $M=\sqrt{\frac{\sigma}{\mu}}B_0d_1$ is the Hartman number, and E_1 $(=-\tau d_1^3/\lambda^3\mu c)$, E_2 $(=mcd_1^3/\lambda^3\mu)$, E_3 $(=dd_1^3/\lambda^2\mu)$, E_4 $(=Bd_1^3/\lambda^3 c\mu)$, E_5 $(=Hd_1^3/\lambda c\mu)$ are the non-dimensional elasticity parameters.

3. Solution Procedure

In order to develop the series solution, we expand ψ, S_{xx}, S_{xy}, S_{yy} about δ [5, 10, 11]:

$$\psi=\psi_0+\delta\psi_1+\delta^2\psi_2+\ldots, \tag{18}$$

$$S_{xx}=S_{0xx}+\delta S_{1xx}+\delta^2 S_{2xx}+\ldots, \tag{19}$$

$$S_{xy}=S_{0xy}+\delta S_{1xy}+\delta^2 S_{2xy}+\ldots, \tag{20}$$

$$S_{yy}=S_{0yy}+\delta S_{1yy}+\delta^2 S_{2yy}+\ldots. \tag{21}$$

3.1. Zeroth-Order System

Upon making use of (18) – (21) into (12) – (17) and choosing the coefficients of δ^0, we have

$$S_{0xx}-2\lambda_1 S_{0xy}\psi_{0yy}=0, \tag{22}$$

$$S_{0xy}-\lambda_1 S_{0yy}\psi_{0yy}=\psi_{0yy}, \tag{23}$$

$$S_{0yy}=0, \tag{24}$$

$$\frac{\partial^2 S_{0xy}}{\partial y^2}-N^2\psi_{0yy}=0, \tag{25}$$

$$\psi_{0y}\pm\beta\psi_{0yy}=0 \quad \text{at } y=\pm\eta, \tag{26}$$

$$\left[E_1\frac{\partial^3}{\partial x^3}+E_2\frac{\partial^3}{\partial x\partial t^2}+E_3\frac{\partial^2}{\partial t\partial x}+E_4\frac{\partial^5}{\partial x^5}+E_5\frac{\partial}{\partial x}\right]\eta=\frac{\partial S_{0xy}}{\partial y}-N^2\psi_{0y} \quad \text{at } y=\pm\eta. \tag{27}$$

Through (22) – (27), the solution is of the form

$$\psi_0=L\left(\frac{\sinh Ny}{N\gamma}-y\right). \tag{28}$$

3.2. First-Order System

The coefficients of $O(\delta)$ give

$$S_{1xx}+\lambda_1\left[\left(\frac{\partial}{\partial t}+\psi_{0y}\frac{\partial}{\partial x}-\psi_{0x}\frac{\partial}{\partial y}\right)S_{0xx}-2(\psi_{0xy}S_{0xx}+\psi_{1yy}S_{0xy}+\psi_{0yy}S_{1xy})\right]=2\psi_{0xy}, \tag{29}$$

$$S_{1xy}+\lambda_1\left[\left(\frac{\partial}{\partial t}+\psi_{0y}\frac{\partial}{\partial x}-\psi_{0x}\frac{\partial}{\partial y}\right)S_{0xy}-(\psi_{1yy}S_{0yy}+\psi_{0yy}S_{1yy})\right]=\psi_{1yy}, \tag{30}$$

$$S_{1yy}+\lambda_1\left[\left(\frac{\partial}{\partial t}+\psi_{0y}\frac{\partial}{\partial x}-\psi_{0x}\frac{\partial}{\partial y}\right)S_{0yy}+2\psi_{0xy}S_{0yy}\right]=-2\psi_{0xy}, \tag{31}$$

$$Re\left(\frac{\partial}{\partial t}+\psi_{0y}\frac{\partial}{\partial x}-\psi_{0x}\frac{\partial}{\partial y}\right)\psi_{0yy}=\lambda_1\left(\frac{\partial}{\partial t}+\psi_{0y}\frac{\partial}{\partial x}-\psi_{0x}\frac{\partial}{\partial y}\right)\left(\frac{\partial^2 S_{0xy}}{\partial y^2}-M^2\psi_{0yy}\right)+\frac{\partial^2}{\partial x\partial y}(S_{0xx}-S_{0yy})-N^2\psi_{1yy}+\frac{\partial^2 S_{1xy}}{\partial y^2}, \tag{32}$$

$$\psi_{1y}\pm\beta\left[\psi_{1yy}-\lambda_1\left(\frac{\partial}{\partial t}+\psi_{0y}\frac{\partial}{\partial x}-\psi_{0x}\frac{\partial}{\partial y}\right)\psi_{0yy}-2\lambda_1\psi_{0yy}\psi_{0xy}\right]=0 \quad \text{at } y=\pm\eta, \tag{33}$$

$$\lambda_1\left(\frac{\partial}{\partial t}+\psi_{0y}\frac{\partial}{\partial x}-\psi_{0x}\frac{\partial}{\partial y}\right)\left[E_1\frac{\partial^3}{\partial x^3}+E_2\frac{\partial^3}{\partial x\partial t^2}+E_3\frac{\partial^2}{\partial x\partial t}+E_4\frac{\partial^5}{\partial x^5}+E_5\frac{\partial}{\partial x}\right]\eta=\left[\frac{\partial S_{0xx}}{\partial x}+\frac{\partial S_{1xy}}{\partial y}+\left(\frac{\partial}{\partial t}+\psi_{0y}\frac{\partial}{\partial x}-\psi_{0x}\frac{\partial}{\partial y}\right)\left(\lambda_1\frac{\partial S_{0xy}}{\partial y}-(\lambda_1M^2+Re)\psi_{0y}\right)-N^2\psi_{1y}\right] \quad \text{at} \quad y=\pm\eta. \tag{34}$$

Substituting the zeroth-order solution into the first-order system and then solving the resulting systems, we obtain

$$\psi_1=A_2y+A_4\sinh Ny+L_2y\cosh Ny+L_3y^2\sinh Ny, \tag{35}$$

in which

$$L=\frac{8\varepsilon\pi^3}{N^2}\left[\frac{E_3}{2\pi}\sin 2\pi(x-t)-\left(E_1+E_2-4\pi^2E_4-\frac{E_5}{4\pi^2}\right)\cos 2\pi(x-t)\right],$$

$$N=\sqrt{M^2+\frac{1}{K}},$$

$$L_1=\frac{NL_t}{\gamma}-\frac{\eta_tN^2L\big(\sinh N\eta+\beta N\cosh N\eta\big)}{\gamma^2},$$

$$\gamma=(\cosh N\eta+\beta N\sinh N\eta),$$

$$L_2=\frac{\big(Re+\lambda_1M^2\big)}{2N^2}\left\{\frac{L_1\,(1+L)}{N}+\frac{5LL_t}{2\gamma}\right\},$$

$$L_3=\frac{\big(Re+\lambda_1M^2\big)\,LL_t}{4N\gamma},$$

$$A_0=\frac{L\left(L_tN^2\cosh N\eta-L_1N\right)}{\gamma},$$

$$A_1=(1+L)\left(\frac{L_1\cosh N\eta}{N}-L_t\right)+\frac{L}{\gamma}\left\{L_t\,(\cosh N\eta-\eta N\sinh N\eta)-\frac{L_1}{N}\right\},$$

$$A_2=\frac{1}{N^2}\left\{2L_2N^2\cosh N\eta+L_3\{4N^2\eta\sinh N\eta+6N\cosh N\eta\}-\lambda_1N^2L_t\left(1+L-\frac{L\cosh N\eta}{\gamma}\right)-\lambda_1A_0-\big(Re+\lambda_1M^2\big)\,A_1\right\},$$

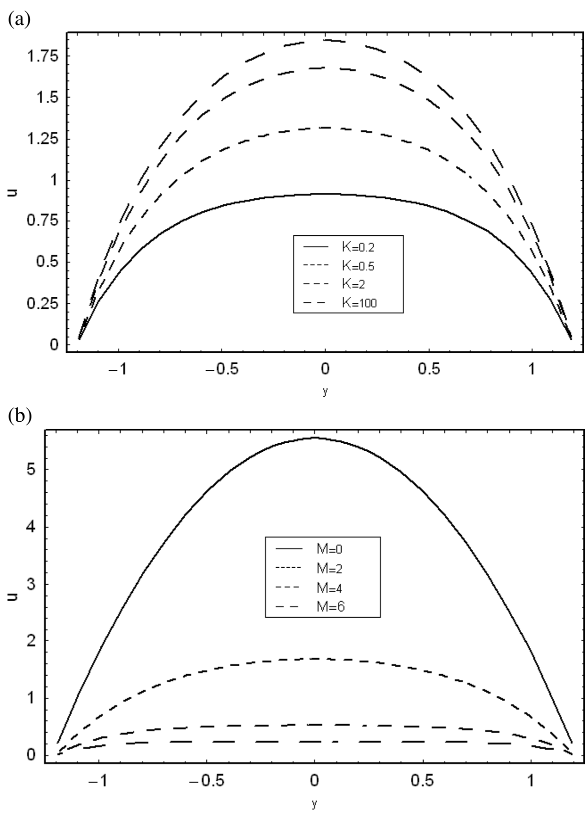
$$A_3=\left\{L_1\,(1+L)+\frac{2NL}{\gamma}\left(L_t-\frac{L_1\cosh N\eta}{N}\right)\right\}\sinh N\eta-\frac{N^2LL_t\eta}{\gamma}\cosh N\eta,$$

$$A_4=\frac{1}{N\gamma}\Big\{-A_2+\beta\lambda_1A_3+(\beta NL_2-\eta L_3)\big(2\sinh N\eta+N\eta\cosh N\eta\big)\,(4\beta L_3\eta N-L_2)\cosh N\eta+\left[\beta L_3\left(\eta^2N^2+2\right)-L_2\eta N\right]\sinh N\eta\Big\},$$

and subscripts denote the partial derivatives.

4. Discussion

The purpose of this section is to predict the variation of emerging parameters on the axial velocity $u=u_0+\delta u_1=\psi_{0y}+\delta\psi_{1y}$ and the stream function ψ. In particular, the variation of permeability parameter K, the Hartman number M, the elastic tension, i. e., E_1 the elastic tension in the membrane, E_2 the mass per unit area, E_3 the coefficient of viscous damping, E_4 the flexural rigidity of the plate, E_5 the spring stiffness, the wave number δ, the relaxation parameter λ_1, Reynolds number Re, and slip parameter β are examined. Figure 1 depicts the behaviour of parameters involved in the axial velocity. It is obvious from Figure 1a that the velocity increases when K increases. Figure 1b shows that u decreases by increasing M. By Figures 1c – f, we



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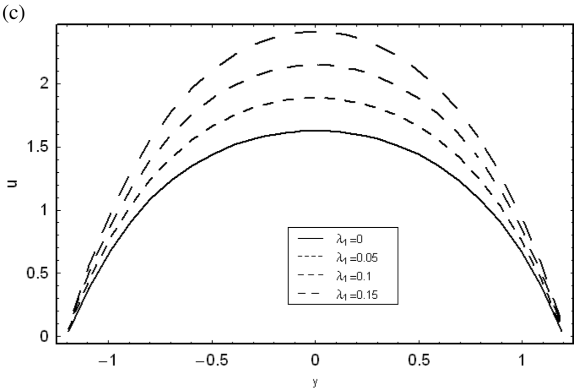

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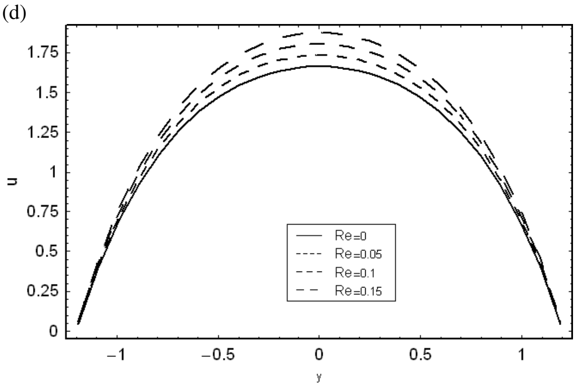

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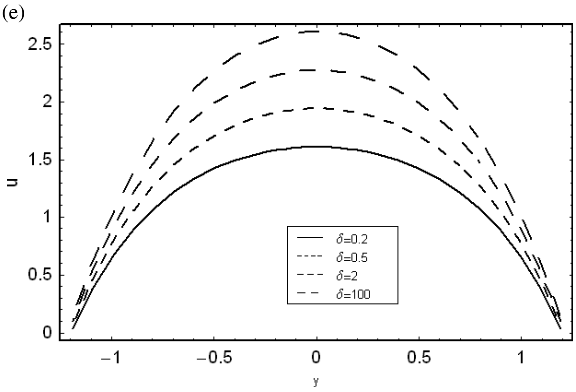

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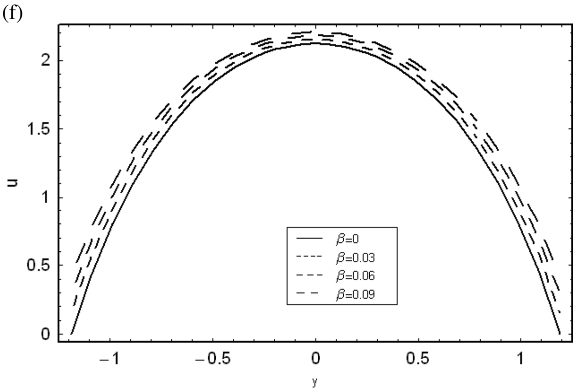

(a)

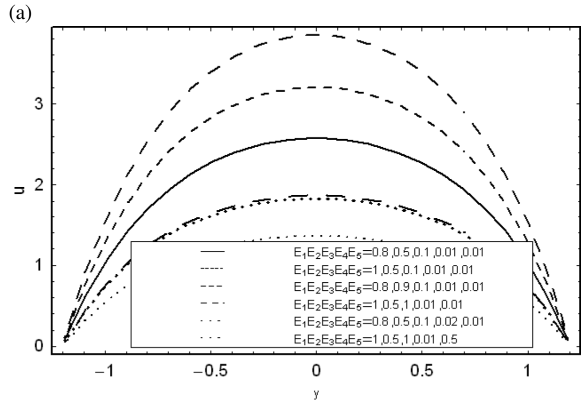

Fig. 1. (a) Variation of K on u when $E_1 = 1$; $E_2 = 0.2$; $E_3 = 0.5$; $E_4 = 0.01$; $E_5 = 0.01$; $\varepsilon = 0.2$; $M = 2$; $x = 0.3$; $t = 0.1$; $Re = 0.01$; $\delta = 0.01$; $\lambda_1 = 0.01$; $\beta = 0.01$. (b) Variation of M on u when $E_1 = 1$; $E_2 = 0.2$; $E_3 = 0.5$; $E_4 = 0.01$; $E_5 = 0.01$; $\varepsilon = 0.2$; $K = 2$; $x = 0.3$; $t = 0.1$; $Re = 0.01$; $\delta = 0.01$; $\lambda_1 = 0.01$; $\beta = 0.01$. (c) Variation of λ_1 when $E_1 = 1$; $E_2 = 0.2$; $E_3 = 0.5$; $E_4 = 0.01$; $E_5 = 0.01$; $\varepsilon = 0.2$; $K = 2$; $M = 2$; $x = 0.3$; $t = 0.1$; $Re = 0.01$; $\delta = 0.01$; $\beta = 0.01$. (d) Variation of Re when $E_1 = 1$; $E_2 = 0.2$; $E_3 = 0.5$; $E_4 = 0.01$; $E_5 = 0.01$; $\varepsilon = 0.2$; $K = 2$; $M = 2$; $x = 0.3$; $t = 0.1$; $\delta = 0.01$; $\lambda_1 = 0.01$; $\beta = 0.01$. (e) Variation of δ when $E_1 = 1$; $E_2 = 0.2$; $E_3 = 0.5$; $E_4 = 0.01$; $E_5 = 0.01$; $\varepsilon = 0.2$; $K = 2$; $M = 2$; $x = 0.3$; $t = 0.1$; $Re = 0.01$; $\lambda_1 = 0.01$; $\beta = 0.01$. (f) Variation of β when $E_1 = 1$; $E_2 = 0.2$; $E_3 = 0.5$; $E_4 = 0.01$; $E_5 = 0.01$; $\varepsilon = 0.2$; $K = 2$; $M = 2$; $x = 0.3$; $t = 0.1$; $Re = 0.01$; $\delta = 0.01$; $\lambda_1 = 0.01$. (g) Variation of compliant wall parameters when $K = 2$; $\varepsilon = 0.2$; $M = 2$; $x = 0.3$; $t = 0.1$; $Re = 0.01$; $\delta = 0.01$; $\lambda_1 = 0.01$; $\beta = 0.01$.

note that the velocity increases when there is an increase of the relaxation parameter λ_1, Reynolds number Re, δ, and the slip parameter β. The compliant wall effects E_1, E_2, E_3, E_4, and E_5 on the velocity are sketched in Figure 1g. It is shown that the velocity increases with an increase of E_1 and E_2. The velocity decreases for an increase in E_3, E_4, and E_5.

The formulation of an internally circulating bolus of the fluid by closed streamlines is called trapping. Figures 2 – 7 aim to examine the trapped bolus. Figure 2 shows the variation of the permeability parameter K on the streamlines. Here the size of trapped bolus increases with an increase in K. The trapped bolus decreases when M increases (Fig. 3). Figure 4 shows the behaviour of λ_1. It depicts that the size of the bolus increases with an increase in λ_1. The size of the trapped bolus increases with the increase of δ (Fig. 5). Figure 6 indicates that there is a very small increase in the size of the trapped bolus when β increases but there is no such effect appearing for the Reynolds number. We analyzed in Figure 7 that the size of the trapped

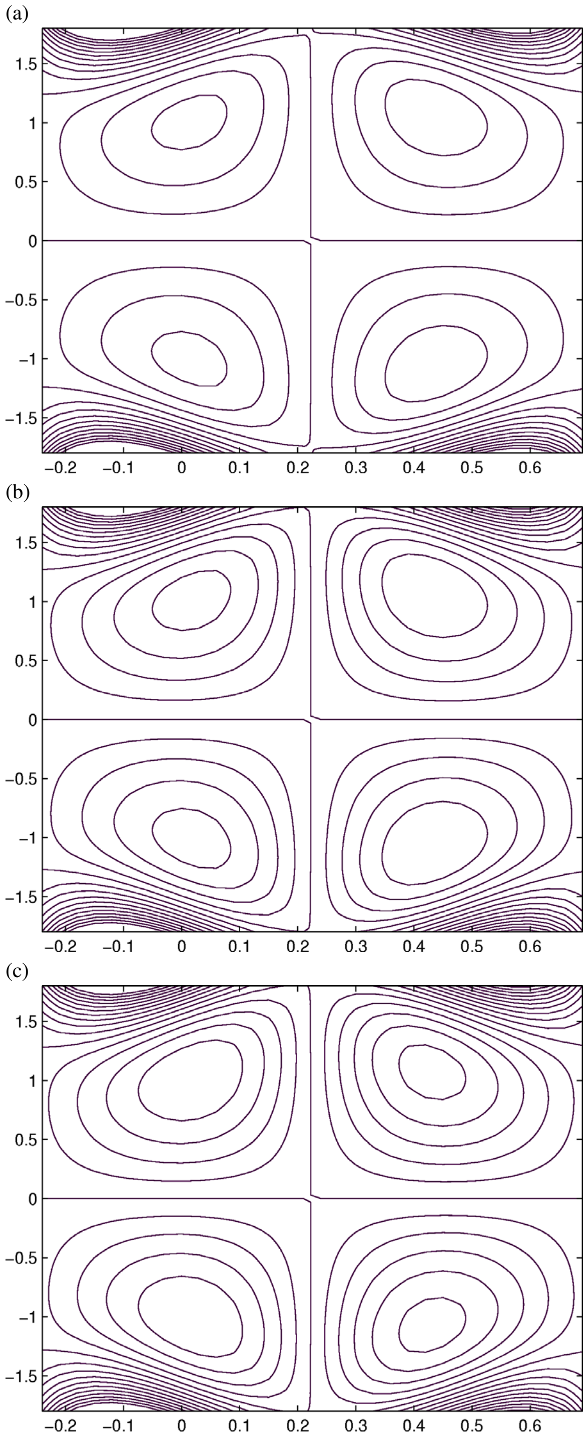

Fig. 2. Streamlines for $E_1 = 0.2$; $E_2 = 0.4$; $E_3 = 0.2$; $E_4 = 0.01$; $E_5 = 1$; $\varepsilon = 0.2$; $M = 2$; $\delta = 0.01$; $t = 1$; $Re = 0.01$; $\lambda_1 = 0.01$; $\beta = 0.01$; (a) $K = 0.2$; (b) $K = 0.5$; (c) $K = 0.8$.

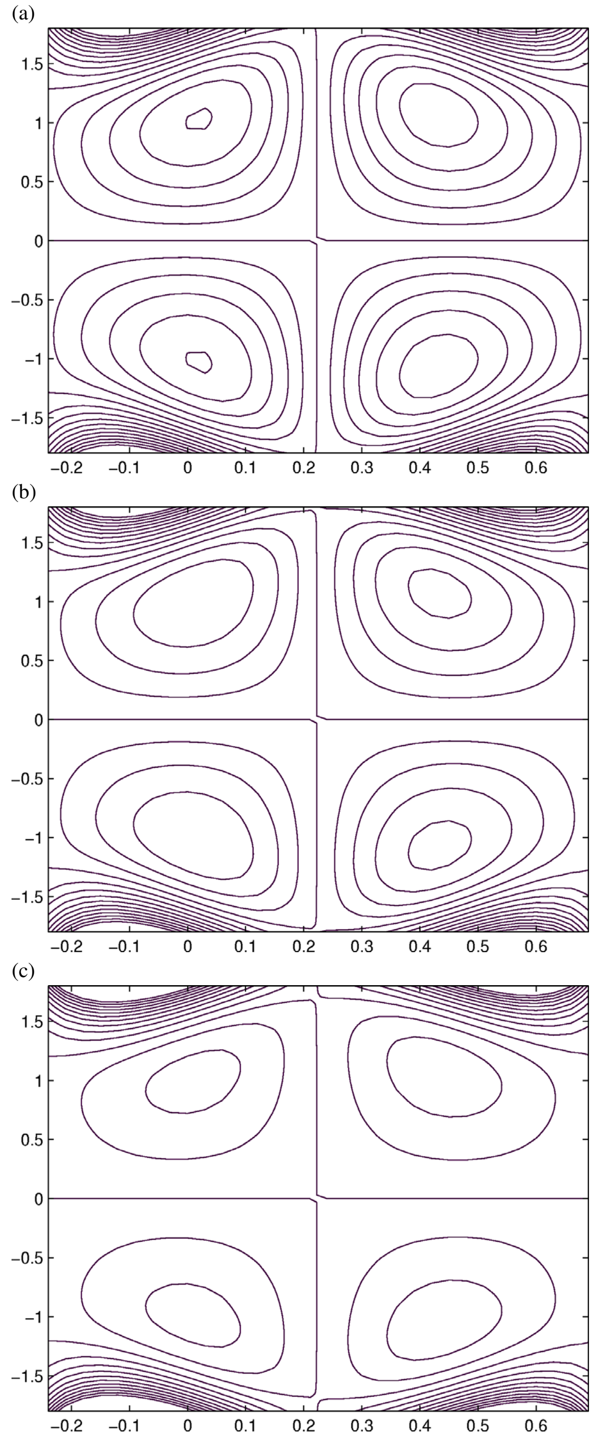

Fig. 3. Streamlines for $E_1 = 0.2$; $E_2 = 0.4$; $E_3 = 0.2$; $E_4 = 0.01$; $E_5 = 1$; $\varepsilon = 0.2$; $K = 0.2$; $t = 1$; $\delta = 0.01$; $Re = 0.01$; $\lambda_1 = 0.01$; $\beta = 0.01$; (a) $M = 0$; (b) $M = 1.5$; (c) $M = 3$.

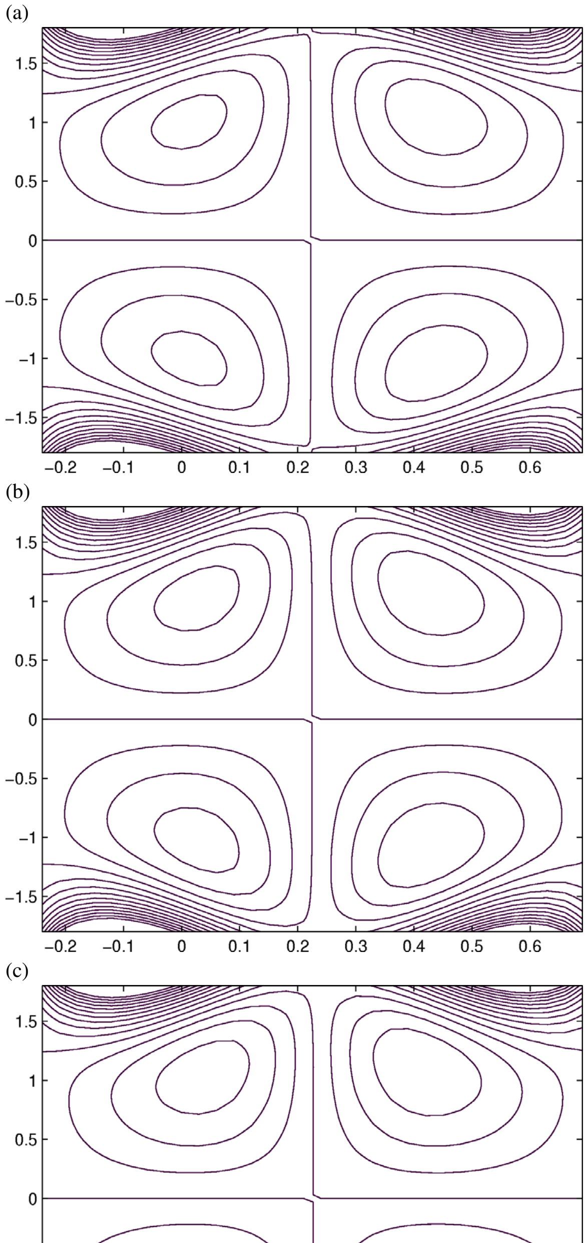

Fig. 4. Streamlines for $E_1 = 0.2$; $E_2 = 0.4$; $E_3 = 0.2$; $E_4 = 0.01$; $E_5 = 1$; $\varepsilon = 0.2$; $M = 2$; $K = 0.2$; $t = 1$; $Re = 0.01$; $\delta = 0.01$; $\beta = 0.01$; (a) $\lambda_1 = 0$; (b) $\lambda_1 = 0.5$; (c) $\lambda_1 = 1$.

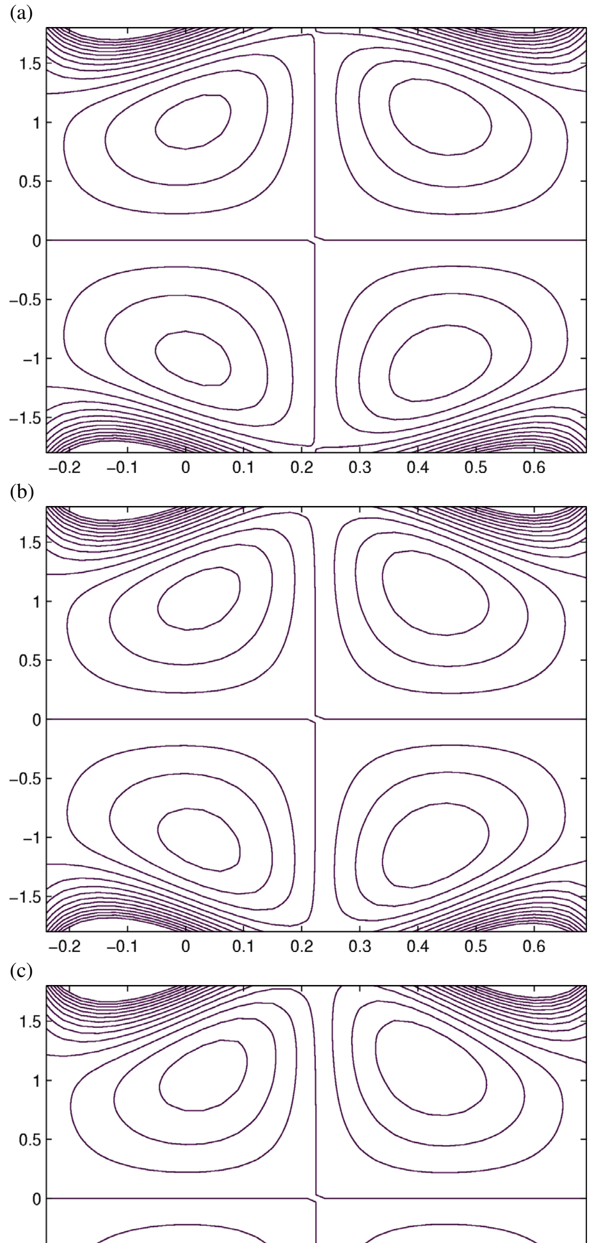

Fig. 5. Streamlines for $E_1 = 0.2$; $E_2 = 0.4$; $E_3 = 0.2$; $E_4 = 0.01$; $E_5 = 1$; $\varepsilon = 0.2$; $M = 2$; $K = 0.2$; $t = 1$; $Re = 0.01$; $\lambda_1 = 0.01$; $\beta = 0.01$; (a) $\delta = 0$; (b) $\delta = 0.3$; (c) $\delta = 0.6$.

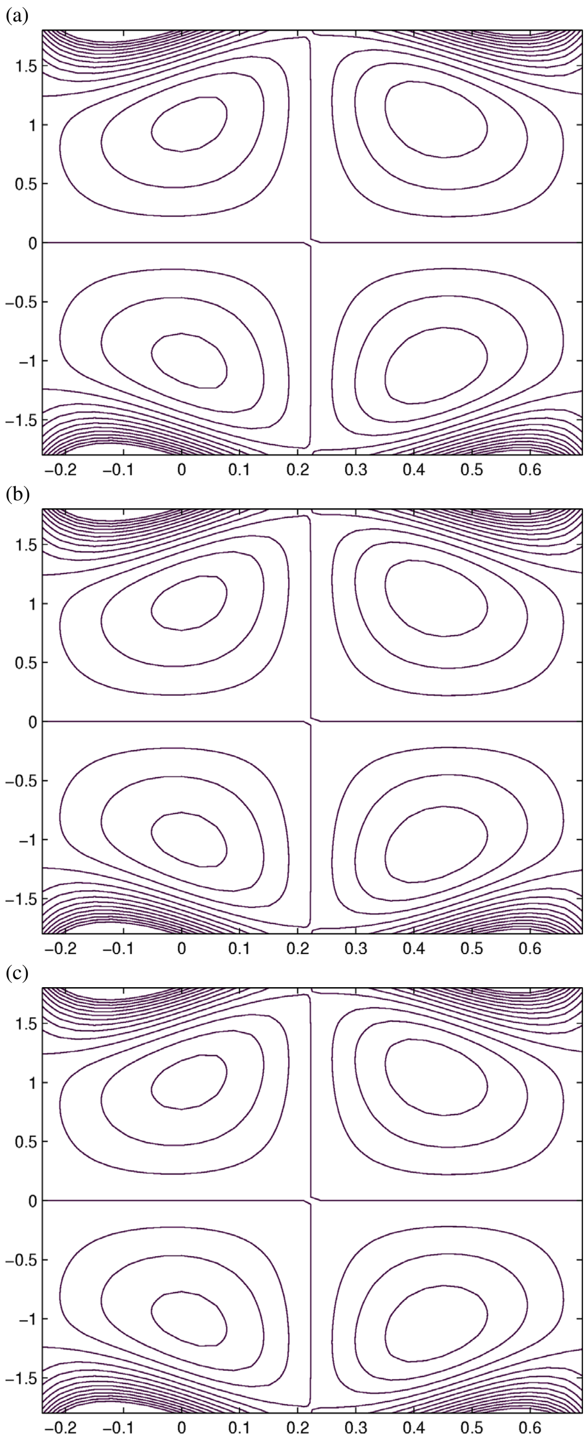

Fig. 6. Streamlines for $E_1 = 0.2$; $E_2 = 0.4$; $E_3 = 0.2$; $E_4 = 0.01$; $E_5 = 1$; $M = 2$; $K = 0.04$; $\varepsilon = 0.2$; $t = 1$; $Re = 0.01$; $\delta = 0.01$; $\lambda_1 = 0.01$; (a) $\beta = 0$; (b) $\beta = 1$; (c) $\beta = 2$.

bolus increases by increasing E_1, E_2, E_3 and decreases by increasing E_4 and E_5.

5. Conclusions

In this study, the slip effects on the MHD flow of a Maxwell fluid are discussed. The main points of presented analysis are as follows.

- By increasing K, E_1, E_2, β, δ, Re, and λ_1, the axial velocity increases.
- The velocity decreases when M, E_3, E_4, and E_5 are increased.
- The role of K and M on the size of the trapped bolus are opposite. The size of trapped bolus increases by increasing K and β.
- The trapped bolus is found insensitive with respect to Re.
- The size of trapped bolus increases by increasing E_1, E_2, E_3, λ_1, and δ. It decreases when $E_4$and E_5 are increased.

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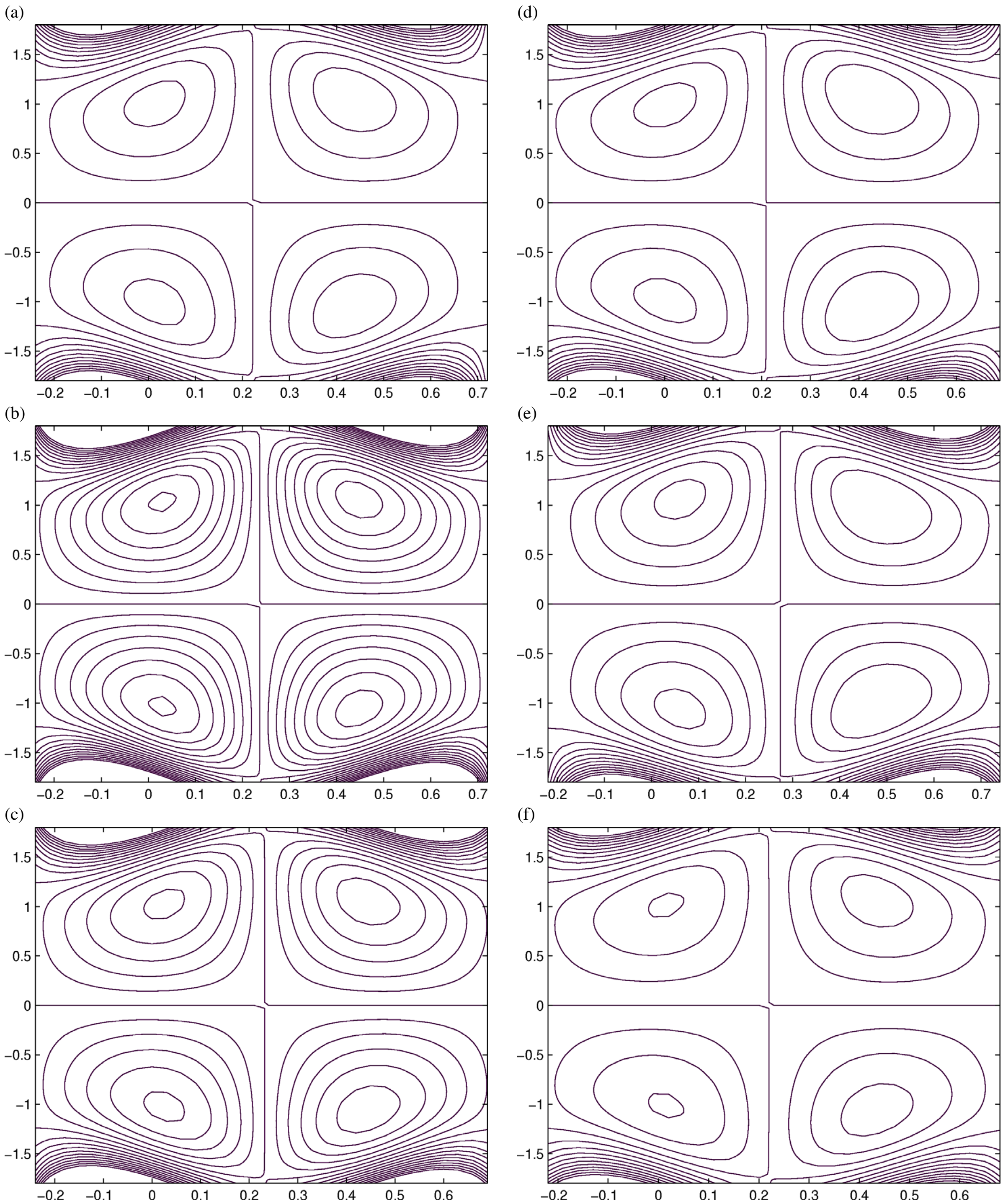

Fig. 7. Streamlines for $\varepsilon = 0.2$; $M = 2$; $K = 0.2$; $t = 1$; $Re = 0.01$; $\delta = 0.01$; $\lambda_1 = 0.01$; $\beta = 0.01$; (a) $E_1 = 0.2$; $E_2 = 0.4$; $E_3 = 0.2$; $E_4 = 0.01$; $E_5 = 1$; (b) $E_1 = 0.4$; $E_2 = 0.4$; $E_3 = 0.2$; $E_4 = 0.01$; $E_5 = 1$; (c) $E_1 = 0.2$; $E_2 = 0.5$; $E_3 = 0.2$; $E_4 = 0.01$; $E_5 = 1$; (d) $E_1 = 0.2$; $E_2 = 0.4$; $E_3 = 0.3$; $E_4 = 0.01$; $E_5 = 1$; (e) $E_1 = 0.2$; $E_2 = 0.4$; $E_3 = 0.2$; $E_4 = 0.02$; $E_5 = 1$; (f) $E_1 = 0.2$; $E_2 = 0.4$; $E_3 = 0.2$; $E_4 = 0.01$; $E_5 = 1.5$.